國立中央大學九十學年度轉學生入學試題

物理學系　三年級　科目：應用數學　共 一 頁　第 1 頁

1. [5分]　$Z = 1 + i$, $\sqrt[4]{Z} = ?$

2. [10分]　Legendre polynomials $P_n(x)$ are defined over interval $1 \geq x \geq -1$

   Shows that $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 - 1)/2$ are orthogonal.

   Norm of P_n's is defined as $\| P_n \| = \sqrt{\int_{-1}^{1} P_n^2(x)dx}$

3. [10分]　Find the solution of $y' = =2xy$.

4. [5分]　$\vec{a} = 4\vec{i} - \vec{k}$, $\vec{b} = -2\vec{i} + \vec{j} + 3\vec{k}$, $\vec{a} \times \vec{b} = ?$

5. [20分]　Evaluate $\int_C (x^2 + y^2 + z^2)^2 \, ds$　where C is the arc of the circular helix

   $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 3t\vec{k}$ from A(1, 0, 0) to B(1, 0, 6π).

6. [20分]　Solve the following initial value problem

   $y'' - 2y' + y = 2x^2 - 8x + 4$,　$y(0) = 3$, $y'(0) = 3$.

7. [20分]　One dimensional heat flow is governed by the equation $\partial u/\partial t = c^2 \, \partial^2 u/\partial x^2$

   Find u(x,t), with conditions u(0,t)=0, u(l,t)=0 and u(x,0)=f(x)

8. [10分]　$f(z) = \frac{1}{z^2 - 1}$, find its singular points and the residues at those points

參考用